$\cos\theta = \cos\lambda$ of the given equation is at least as great as the number of coincident points in which the circle cuts the ellipse at the given point. [How the order may be greater than this number will appear shortly.]

The pole being any point whatever on the normal, the circle is tangent to the ellipse, since the two curves have a common normal. There are consequently at least two coincident points of intersection.

If the pole be at the center of curvature for the given normal, the circle is the osculating circle, which cuts the ellipse in at least three coincident points.

If the pole be on the major axis, then it follows from the symmetry that the circle is tangent to the ellipse at two points, where $\theta = \lambda$ and where $\theta = -\lambda$. But since $\cos(-\lambda) = \cos\lambda$, it follows that for all four intersections $\cos\theta = \cos\lambda$, so that this solution is of multiplicity four.

MECHANICS.

220. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Four particles A, B, C, D, lie on a smooth table at the corners of a rhombus. AB, BC, CD, DA are light inextensible strings connecting the particles. The angle at A is acute. A blow is given to A along the diagonal, away from C. Find the ratio of the initial velocity of C to that of A.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let v=velocity of A. Then since the table and particles are smooth the component of v along the direction BA is $y = v\cos\frac{1}{2}A$. The component of y in the direction CB is $z = y\cos A = v\cos A\cos\frac{1}{2}A$. The component of z in the direction CA is $u = z\cos\frac{1}{2}A$.

$\therefore u = v\cos A\cos^2\frac{1}{2}A$. But u is the initial velocity of C.

$\therefore v : u = 1 : \cos A\cos^2\frac{1}{2}A$.

As $\angle A$ decreases every instant, u increases every instant until $\angle A = 0^\circ$, $u = v$.

221. Proposed by W. J. GREENSTREET, Stroud, England.

Two smooth intersecting planes are each at 45° to the horizon. Between them lies a cylinder of elliptic cross section. Find the position of equilibrium.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let P, Q be the points of contact of the cylinder with the planes, intersecting at an angle of 90°. Let the normals at P, Q intersect in O and let C be the center of an elliptic section in the same plane as OP, OQ. Then either C and O coincide or CO is vertical. If they coincide the axes of the elliptic section are parallel to the planes. This gives one position of equilibrium. When OC is vertical, OC makes an angle of 45° with both planes.

From the force diagram the reactions at P, Q are equal, hence either the major axis is vertical or the minor axis is vertical, giving the other two positions of equilibrium.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

156. Proposed by A. H. HOLMES, Brunswick, Maine.

Find integral values for a, b, c, d, and e in the equation, $a^2+b^2+c^2+d^2=e^2$.

I. Solution by the PROPOSER.

Take $a=32(40^2-9^2)$; $b=64(40\times9)$; $c=24(40^2+9^2)$; $d=9(40^2+9^2)$. Then $a^2=32^2\times1519^2$; $b^2=32^2\times720^2$; $c^2=24^2\times1681^2$; $d^2=81\times1681^2$. And $e^3=32^2[1519^2+720^2+1681^2(24^2+81)]$.

$\therefore a=48608$; $b=23040$; $c=40344$; $d=15129$; $e=1681$.

In the above problem it was intended by the author that e^2 should be e^3. ED. F.

II. Solution by ARTEMAS MARTIN, LL. D., Coast Survey Office, Washington, D. C.

In the well-known identity

$$(x-y)^2+4xy=(x+y)^2, \tag{1}$$

x and y may have any values whatever, and we can, therefore, assume $x=u+v+w$; then (1) becomes

$$(u+v+w-y)^2+4y(u+v+w)=(u+v+w+y)^2. \tag{2}$$

Now take $u=p^2$, $v=q^2$, $w=r^2$, $y=s^2$; then (2) becomes

$$(p^2+q^2+r^2-s^2)^2+(2ps)^2+(2qs)^2+(2rs)^2=(p^2+q^2+r^2+s^2)^2, \quad (3);$$

and p, q, r, s may be any values chosen at pleasure.

Therefore we may take $a=p^2+q^2+r^2+s^2$, $b=2ps$, $c=2qs$, $d=2rs$, $e=p^2+q^2+r^2+s^2$, or in any other order we please.

Examples. 1. Take $p=1$, $q=2$, $r=3$, $s=1$; then $2^2+4^2+6^2+13^2=15^2$.

2. Take $p=1$, $q=2$, $r=3$, $s=4$; then, after dividing through by 2^2 and discarding the negative sign, $1^2+4^2+8^2+12^2=15^2$.

3. Take $p=2$, $q=3$, $r=4$, $s=5$; then, after dividing through by 2^2, $2^2+10^2+15^2+20^2=27^2$.

See *Mathematical Magazine*, Vol. II, No. 5, pp. 69-76; No. 6, pp. 89-96; No. 8, pp. 137-140; and No. 11, pp. 209-220, for various methods of finding many sets of square numbers whose sum is a square.